# Expected value and decision making under conditions of uncertainty

By Simon White SC*

The purpose of this article is to explain the importance of expected value (also known as mathematical expectation) in the context of decision making under conditions of uncertainty. Whether it is a game of cards, horse racing, investing, litigation or any other field of human endeavour involving probabilistic outcomes an optimal decision cannot be made unless the decision maker has regard to expected value.

Expected value is both a qualitative and a quantitative approach to decision making where the outcome is uncertain. It requires the decision maker to consider the following. First, the range of expected outcomes if a decision is made. Secondly, allocating to each of the expected outcomes a probability (in percentage terms) which reflects the decision maker's belief as to the chances of that outcome in fact occurring. Thirdly, multiplying the probability by the expected outcome which identifies the expected payoff for that particular outcome. Fourthly, adding the various expected payoffs to determine expected value which may be positive or negative.

Approaching decision making in this way requires the decision maker to focus not merely on probable outcomes (the frequency of the outcome) but the expected value (the frequency of the outcome multiplied by the payoff). This provides the decision maker with a better understanding of the upside/downside potential when making the decision.

Prior to discussing expected value in the context of litigation a brief history of its discovery is set out below. Perhaps unsurprisingly it was the context of games of chance that expected value was conceived.

Until the mid-seventeenth century the problem of the points also known as the problem of division of the stakes remained without a satisfactory answer. Its resolution by two Frenchmen in a series of letters commencing in 1654 created for the first time a theory of probability that was capable of mathematical solution.

The problem of the points concerns a game of chance with two players who have equal chances of winning each round. The players contribute equally to a prize pot and agree in advance the first player to have won a certain number of rounds will collect the entire prize. Now suppose the game is interrupted by external circumstances before either player has achieved victory. How does one divide the pot fairly?

Luca Pacioli (c1447–1517) an Italian mathematician and the first to publish a work on double entry system of book keeping (hence referred to as the father of accounting) considered the problem of the points in 1494. He proposed the answer was to divide the stakes in proportion to the number of rounds won by each player[1]. The number of rounds needed to win did not enter Pacioli's calculations.

In the mid-sixteenth century Niccolo Tartagia (1500–1557) observed that according to Pacioli's method if the game was interrupted when only one round had been played the entire pot would be awarded to the winner of the single round albeit a one round lead in a long game was far from decisive. Tartagia was unsure whether the problem was soluble at all in a way that would convince both players of its fairness: 'In whatever way the decision is made there will be a cause for litigation'[2].

One hundred years after Tartagia's pessimistic observation (albeit perhaps not to the ears of a lawyer) Blaise Pascal (1623–1662) and Pierre de Fermat (c1601–1665) resolved the problem of the points. Pascal was amongst other things a mathematician, inventor and the author of a note that became known as Pascal's Wager. Fermat was a French lawyer, mathematician and credited with contributing to the early development of calculus.

Following their introduction in 1654 Pascal and Fermat discussed in a series of letters the problem of the points. Their solution laid the groundwork for the theory of probability.[3] Pascal and Fermat constructed a systematic method for analysing future outcomes. They provided a procedure for determining the likelihood of each of the possible outcomes assuming the outcomes could be measured mathematically.

The insight of Pascal and Fermat was that the division of the pot should depend not so much on the history of the game to the time of interruption but on the possible ways the game might have continued were it not interrupted. As stated by Pascal in a letter to Fermat: '…the rule determining that which will belong to them [when the game is terminated] will be proportional to that which they had the right to expect from fortune'[4]. In other words the value of a future gain should be directly proportional to the chance of getting it.

John Maynard Keynes in his *Treatise on Probability* published in 1920 states that mathematical expectation represents the product of the possible gain with the probability of attaining it[5]. He states:

> In order to obtain, therefore, a measure of what ought to be our preference in regard to various alternative courses of action, we must sum for each course of action a series of terms made up of the amounts of good which may attach to each of its possible consequences, each multiplied by its appropriate probability.[6]

Keynes considered the conception of mathematical expectation could be claimed by Gottfried Leibniz (1646–1716) based on a 1678 publication. Whilst Keynes refers in the bibliography appearing in his Treatise to the letters passing between Pascal and Fermat he dates them also as at 1678. Relevantly for our purposes Keynes refers to a letter from Leibniz to Vincent Placcius (1642–1699) dated 1687 in which he applied mathematical expectation to jurisprudence. The letter gives an example of two litigants who lay claim to a sum of money, and if the claim of one is twice as probable as that of the other, the sum should be divided between them in that proportion. Keynes notes that whilst the doctrine seems sensible 'I am not aware it has ever been acted on'.

Whilst Pascal and Fermat were concerned with the problem of the points, their solution transcends gambling and constitutes a framework which can be used in situations that involve decision making under conditions of uncertainty. Litigation is an example par excellence that requires decisions to be made under such conditions.

All decisions involve the weighing of probabilities. This involves (as found by Pascal and Fermat) balancing the probability of an outcome (frequency) with the outcome's payoff (magnitude). There are, however, two types of probabilistic decisions. The first is one in which the probability of the outcome (frequency) and the outcome's payoff (magnitude) are symmetrical. For example two persons (A and B) bet each other $1, even money, on the flip of a coin. Each time it comes up heads A wins (B loses) and each time it comes up tails B wins (A loses). In this example the expected value is zero because the probability of gain by the outcome (0.5 x $1) minus the probability of loss by the outcome (0.5 x $1) equals zero. This is not to say that after a number of tosses A will not be ahead of B or vice versa. Expected value is the mathematical amount a bet will average winning or losing. It has nothing to do with results. Player A might win five tosses in a row but in the long run the tosses will reflect the sum of the players' expectations.

The second scenario, however, occurs in situations where the probability and the payoff are skewed or asymmetrical. By way of example, returning to the coin game referred to above, let us assume A is willing to bet $2 to B's $1 on the flip of the coin. Now there is asymmetry between probability and outcome which also gives rise to positive expected value (for B) because the probability of gain by the outcome ($2 x 0.5) minus the probability of loss by the outcome (0.5 x $1) is 50 cents.[7] This is an example of asymmetric outcomes (the probability remains constant in both games). Of course one can also have asymmetric probabilities where the probabilities are not 50 per cent for each event but the probability on one side is higher than the probability on the other.

The failure to differentiate between probability and expectation (probability x payoff) can lead to poor decision making. Some high probability propositions are unattractive and some low probability propositions are very attractive on an expected value basis. An example of the former is as follows. A gamble has a 999 chance in 1,000 of making me $1 (event A) and 1 chance in 1,000 of losing me $10,000 (event B). Should I take the bet? If I consider the probability only and ignore expected value (probability x outcome) then the gamble seems a sure winner. However, closer analysis says otherwise. The expectation of event A is about $1(999/1,000 x $1) and the expectation of event B is -$10 (1/1,000 x -$10,000) being a total expected value of about -$9.[8]

*The failure to differentiate between probability and expectation (probability x payoff) can lead to poor decision making.*

In a world where there are few if any provable certainties the key to reaching the best decision is to identify all possible outcomes and decide what odds to attach to each. This involves an ability to estimate probabilities, which in turn depends in part on the range and nature of potential outcomes.

Litigation deals largely with uncertainty whereas gambling in a casino deals largely with risk. In each case the outcomes are unknown but in the case of uncertainty the underlying distribution of outcomes is undefined, while with risk we know what the distribution looks like (for example a dice has a one in six chance of landing on a three).[9] In the context of advising a client in relation to the potential outcome of litigation it requires the lawyer to carefully consider the range of outcomes and pay regard to the degrees of uncertainty that attach to each. This in turn enables the lawyer to advise the client as to the outcome in quantified terms.

The ability to advise a client in quantified terms, namely, a percentage is important for a number of reasons. First, it compels the lawyer to go through the process of identifying the range of possible outcomes and attach probabilities to each. Whilst this process may be difficult and uncertain and some may argue artificial it is far better that the alternative. To advise a client that she has 'reasonable prospects' or 'arguable prospects' or 'poor prospects' is of little utility. What do such vague notions mean and how are they to be understood by the client? Unless the advice conveys the numerical probability of risk there is the prospect the client may attach a different probability range to the verbal label than what was intended.[10] Presumably the lawyer attaches some percentage or range to the words 'reasonable prospects' and if so would it not be appropriate to inform the client of that matter thereby providing the client with information that is clear and comprehensible?

Secondly, if a client is advised she has 'reasonable prospects' of being awarded damages of $100,000 she is likely to make decisions including whether or not to commence or continue litigation on a false premise. That is because the client is unable to determine based on the advice what is the expected value of the litigation. If, however, the client had been advised her chances of being awarded $100,000 was 60 per cent then the lawyer and client could discuss the merits or otherwise of commencing or continuing the litigation on the basis the expected value was $60,000. Such matters are also relevant to the issue of settlement. By considering risk in this way the client can approach settlement with the knowledge of what her case is worth in terms of expected value and use that as the benchmark in settlement discussions.

Thus when advising a client whether to commence litigation the lawyer needs to have an understanding of the range of possible outcomes and attach probabilities to each. If the advice to the client is that she has a 70 per cent probability in relation to a claim for $100,000 then the expected value is $70,000 (EV= 0.7 x $100,000) - (0.3 x $0). However, in most jurisdictions in this country legal costs follow the event. Thus the expected value must take into account the risk the client may be required to pay her own costs and that of the defendant. Therefore the lawyer needs to estimate the costs of the client and those of the defendant. If the lawyer assumes the costs for each party are $20,000 the calculation appears as follows: EV= (0.7 x $120,000) – (0.3 x $40,000) being $72,000. An advice in terms of expected value of $72,000 enables the client to consider whether the litigation is worth the time, stress and anxiety and will also enable the client to give meaningful consideration to any offers of settlement.

From the perspective of the defendant let us assume she has been advised there is a 60 per cent chance of defending the litigation referred to above (and therefore a 60 per cent chance of having her costs of $20,000 paid by the plaintiff) but that if she loses she will be required to pay the damages of $100,000 and her costs and those of the plaintiff totalling $40,000. The expected value is $44,000 because EV= (0.4 x $140,000) – (0.6 x $20,000). To advise a defendant in the above example that she has 'reasonable prospects' does not enable the client to have an understanding of the risk of exposure, namely, $44,000. Expected value seeks to better inform the client of the risk even when the probability is that she may win.

The above example assumed the outcome was either an award of $100,000 or $0. However, it is often the case the quantum of damages that may be awarded is itself uncertain. In such circumstances expected value can assist the client in having a better understanding of the likely outcome should she establish liability. Let us assume the lawyer for the plaintiff in the above example, having regard to her experience, the legal principles and the facts, considers the range of damages in the event her client establishes liability to be between $100,000 and $25,000. The lawyer might then consider the various outcomes as having the following probabilities: $100,000 x 0.50; $75,000 x 0.30; $50,000 x 0.20 and $25,000 x 0.10. This gives an expected value of $85,000. By analysing the range of potential outcomes in this way the client is clearly in a better position to weigh up the risks of commencing or continuing the litigation.

In the event litigation is commenced and as new facts come to light the lawyer can increase or reduce the probabilities attaching to the various outcomes as the circumstances require. Expected value is not static and must reflect new information as it comes to hand.

Thinking of litigation and its outcome in terms of expected value is an important means by which parties can have a better understanding of the risks attaching to the uncertainty of litigation. It enables a client to consider the chances of success and the financial exposure in a rigorous and disciplined way which can only assist in the making of decisions under conditions of uncertainty.

## Endnotes

*The author would like to thank Antony Whitlam QC and Daniel Tynan for their helpful comments in relation to the draft of this paper.


1. VJ Katz, *A History of Mathematics*, 1993, Section 11.3.1.
2. Tantagia, quoted by Katz, op cit.
3. For those interested, the full text of the correspondence, translated into English, appears in David, Florence Nightingale, 1962, *Games, Gods and Gambling*, Hefner Publishing Co.
4. P Bernstein, *The Remarkable Story of Risk*, 1998 p67.
5. Ibid., p311.
6. Ibid.
7. This example and the application of expected value in the context of poker appears in Sklansky, *The Theory of Poker*, 1999, Chapter 2.
8. For this example and expected value in the context of share trading see Nassim Taleb, *Fooled by Randomness*, 2005, Chapter 6. For the application of expected value in the context of horse racing see *Crist on Value* by Steven Crist, Chapter 3 in *Bet with the Best*, 2001.
9. Mauboussin, *More Than You Know, Finding Financial Wisdom in Unconventional Places*, 2008, p11.
10. D Evans, *Risk Intelligence*, 2012, p.117.